Problem Set 1 due February 25, at 10 PM, on Gradescope

Please list all of your sources: collaborators, written materials (other than our textbook and lecture notes) and online materials (other than Gilbert Strang's videos on OCW).

Give complete solutions, providing justifications for every step of the argument. Points will be deducted for insufficient explanation or answers that come out of the blue.

Problem 1:

Find a 3×2 matrix A and a 2×3 matrix B such that:

$$A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2x - y \\ x \\ y \end{bmatrix} \qquad \text{and} \qquad B \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z \\ x \end{bmatrix}$$

for any numbers x, y, z. *(20 points)*

Problem 2:

What is the largest number n of vectors in the xy–plane, such that the dot product of any two of them is **strictly** negative? Argue by exhibiting a collection of n vectors that have this property, and argue why there cannot exist a collection of $n+1$ vectors with this property.

(25 points)

Problem 3:

Consider the matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ e & 1 & 0 \\ \sqrt{2} & \pi & 1 \\ 10^{-1} & 10^{-2} & 10^{-3} \end{bmatrix}$$

and put it in row echelon form. *(25 points)*

Problem 4:

Consider the following system of equations:

$$\begin{cases} x - y + 2z = 1 \\ -2x + \lambda y - z = 0 \\ -3x + 2y - 5z = 0 \end{cases}$$

for some number λ. Answer the following questions about Gaussian elimination for this system.

(1) For which choice of λ does Gaussian elimination require you to swap the second and third rows? Solve the system for that value of λ. *(20 points)*

(2) For which choice of λ is the matrix of the system singular? *(10 points)*